Math 3236 Statistical Theory

4/25/23

Law of Large Number

C.L.T.

Delta methods

Concentration inequalities

phenomena.

Important distributions

Beta, Gamma, χ^2_n, t_n

F_{n_1, n_2}

Multivariate Normal r.v.

Estimation.

$f(x \mid \underline{\theta})$ $\underline{\theta}$ parameters.

X_i $i = 1 \ldots N$

X_i i.i.d. with distribution

$f(x \mid \underline{\theta})$

$\hat{\theta}$ is a estimator

consistent if

$$\hat{\theta} \xrightarrow{P} \theta \quad \text{as} \quad N \to \infty$$

unbiased

$$\mathbb{E}(\hat{\theta}) = \theta$$

Method of Moment

$$\mathbb{E}(X_i) = m(\theta)$$

$$m(\theta) = \bar{X}_N = \frac{1}{N} \sum_i X_i$$

$$\mathbb{E}(X_i(1 - X_i))$$

Est. MM are consistent.

Maximum Likelihook

$$L(\theta \mid \underline{x}) = \prod_{i=1}^{N} f(x_i \mid \theta)$$

$$\ell(\theta \mid \underline{x}) = \sum_{i=1}^{N} \log(f(x_i \mid \theta))$$

$$\arg\sup_{\theta} \ell(\theta \mid x) = \hat{\theta}_{MLE}(\underline{x})$$

$\hat{\theta}_{MLE}(\underline{X})$ estimator.

for N large

$$\hat{\theta}_{MLE}(\underline{X})$$

under reasonable conditions

has a normal distribution, unbiased, consistent, variance = Fisher info

efficient.

Bayesian point of view.

There is a prob. dist. on θ, $\xi(\theta)$. Statistical analysis "consists" in updating the distribution ξ to take into account new data

$$\underbrace{\xi(\theta \mid \underline{x})}_{\text{posterior}} = \frac{f(\underline{x} \mid \theta)\overbrace{\xi(\theta)}^{\text{prior}}}{f(\underline{x})}$$

Congiugated families.

Hyper parameters.

Estimation

$$\xi(\theta)$$

$$L(\alpha, \theta)$$

estimate

true value

$$\hat{\theta} = \arg\min_{\alpha} \mathbb{E}_{\xi}(L(\alpha, \cdot))$$

Confidence Interval.

θ

$$\mathbb{P}\left(A(\underline{X}) \le \theta \le B(\underline{X})\right) \geq \gamma$$

coefficient γ confidence interval.

Pivotal quantities

$r(X, \theta)$ whose distribution

does not depend on θ.

$$X_i \sim N(\mu, \sigma^2)$$

$$\sqrt{N}\left(\frac{\bar{X}_N - \mu}{\sigma}\right) = Z \sim N(0,1)$$

Using the C.L.T we get that

$$\sqrt{N}\left(\frac{\bar{X}_N - \mu}{S}\right) \to N(0,1)$$

sample variance

even if X_i are not Normal

$X_n \Rightarrow X$ in distribution

$S_n \to \sigma$ in probability

$$\frac{X_n}{S_n} \Rightarrow \frac{X}{\sigma} \quad \text{in distribution.}$$

Confidence interval based on the t-distribution and the F distribution.

Test of Hypotheses.

$H_0 : \mu \leq \mu_0$

$H_a : \mu > \mu_0$

T Test statistics

if H_0 is True then you expect T to be negative a small. If T is large it is very unlikely that H_0 is true.

$$X_i \simeq N(\mu, \sigma^2)$$

Known

$$H_0: \mu \leq \mu_0$$

$\overline{X}_N = \frac{1}{N} \sum_i X_i$ not much larger than μ_0.

δ_c : if $\overline{X}_N \geq c$ reject H_0

$\pi(\mu \mid \delta_c)$ = prob that δ_c reject H_0 when true is μ.

size of Test

$$\sup_{\theta \in \Omega_0} \pi(\theta \mid \delta_c) = \alpha$$

T p-value of The Test is The largest α at which you still reject H_0.

The End