

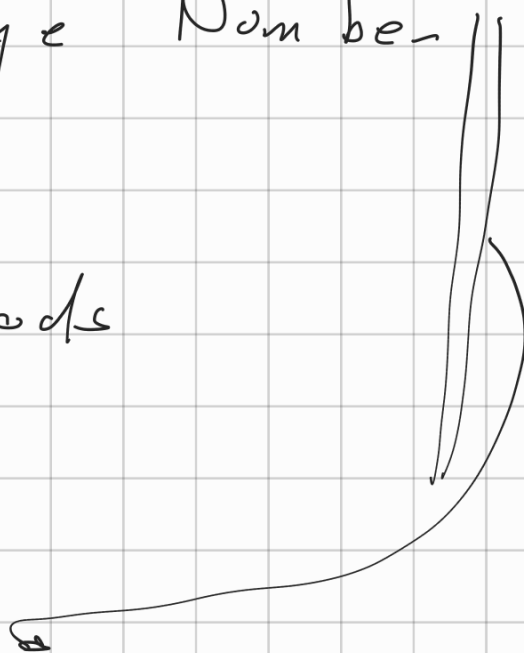
# Math 3236 Statistical Theory

4/25/23

Law of Large Numbers

C.L.T.

Delta methods



Concentration inequalities  
phenomena.

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Important distributions

Beta, Gamma,  $\chi_n^2$ ,  $T_n$

$F_{n_1, n_2}$

Multivariate Normal r.v.

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Estimation.

$f(x|\theta)$   $\theta$  parameters.

$X_i$  i.i.d.  $N$

$X_i$  i.i.d. with distribution

$f(x|\theta)$

$\hat{\theta}$  is an estimator

consistent if

$$\hat{\theta} \xrightarrow{P} \theta \quad \text{as } N \rightarrow \infty$$

unbiased

$$E(\hat{\theta}) = \theta$$

Method of Moment

$$E(X_i) = m(\theta)$$

$$m(\theta) = \bar{X}_N = \frac{1}{N} \sum_i X_i$$

$$E(X_i(1-X_i))$$

Est. MLE are consistent.

Maximum Likelihood

$$L(\theta | \underline{x}) = \prod_{i=1}^N f(x_i | \theta)$$

$$l(\theta | \underline{x}) = \sum_{i=1}^N \log(f(x_i | \theta))$$

$$\arg \sup_{\theta} l(\theta | \underline{x}) = \hat{\theta}_{MLE}(\underline{x})$$

$\hat{\theta}_{MLE}(\underline{x})$  estimator.

For  $N$  large

$\hat{\theta}_{MLE}(\underline{x})$

under reasonable conditions

has a normal distribution, unbiased,

consistent, variance = Fisher info

efficient.

Bayesian point of view.

There is a prob. dist. on  $\theta$ ,  $\xi(\theta)$ . Statistical analysis "consists" in updating the distribution  $\xi$  to take into account new data

$$\xi(\theta | x) = \frac{f(x | \theta) \overset{\text{prior}}{\xi(\theta)}}{f(x)}$$

posterior

Conjugate families.

Hyperparameters.

Estimation

$$\xi(\theta)$$

$$L(a, \theta)$$

estimate  $\uparrow$  True value

$$\hat{\theta} = \underset{a}{\text{arg min}} \mathbb{E}_{\xi} (L(a, \cdot))$$

Confidence Interval.

$\theta$

$$P(A(X) \leq \theta \leq B(X)) \approx \gamma$$

coefficient  $\gamma$  confidence interval.

Pivotal quantities

$v(X, \theta)$  whose distribution

does not depend on  $\theta$ .

$$X_i \approx N(\mu, \sigma^2)$$

$$\sqrt{n} \left( \frac{\bar{X}_n - \mu}{\sigma} \right) = Z \approx N(0, 1)$$

Using the CLT we get that

$$\sqrt{n} \left( \frac{\bar{X}_n - \mu}{s} \right) \rightarrow N(0, 1)$$

sample variance

even if  $X_i$  are not Normal

$X_n \Rightarrow X$  in distribution

$S_n \Rightarrow \sigma$  in probability

$\frac{X_n}{S_n} \Rightarrow \frac{X}{\sigma}$  in distribution.

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Confidence interval based on  
the  $t$ -distribution and the  
 $F$  distribution.

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Test of Hypotheses.

$$H_0 : \mu \leq \mu_0$$

$$H_a : \mu > \mu_0$$

$T$  Test statistics

if  $H_0$  is True then you  
expect  $T$  to be negative

$\sigma$  small. If  $T$  is large  
it is very unlikely that  $H_0$   
is True.

$$X_i \sim N(\mu, \sigma^2)$$

↑  
Known

$$H_0: \mu \leq \mu_0$$

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

not much larger

Then  $\mu_0$ .

$$\delta_c: \text{if } \bar{X}_n \geq c \text{ reject } H_0$$

$\pi(\mu | \delta_c) =$  prob that  $\delta_c$   
reject  $H_0$  when True is  $\mu$ .

size of Test

$$\sup_{\theta \in \Omega_0} \pi(\theta | \delta_c) = \alpha$$

The p-value of the test  
is the largest  $\alpha$  at which  
you still reject  $H_0$ .

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The End